Finite-time interval observer design for discrete-time switched systems: a linear programming approach

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摘要：这篇论文研究了离散切换系统在系统的扰动未知但有界情况下的有限时间有界观测器设计方法，并建立有限时间有界观测器的框架，利用多重线性协同 Lyapunov 函数给出了该观测器的充分条件。此外，用线性规划形式表示的条件可以用标准计算软件进行数值处理。最后通过仿真实例验证了所设计观测器的有效性。

Abstract:This paper deals with the finite-time interval observer design method for discrete-time switched systems subjected to disturbances. The disturbances of the system are unknown but bounded. The framework of the finite-time interval observer is established and the sufficient conditions are derived by multiple linear copositive Lyapunov function. Furthermore, the conditions which are expressed by the forms of linear programming are numerically tractable by standard computing software. An example is simulated to illustrate the validity of the designed observer.

Keywords: Finite-time interval observers, Discrete-time switched systems, Linear programming

Introduction

State estimation is very important since it can be used in stabilization, synchronization, fault diagnosis and detection and so on. As we know, the uncertainties always exist in the real systems. When we design the observers for uncertain systems, the uncertainties should be taken into account. For the purpose of estimation of bounds of the states, the definition of interval observer (IO) was firstly introduced by [1]. Then, the IO design method has been established for a large amount of systems, such as linear systems [2,3], linear parameter varying systems [4,5], singular systems [6,7], discrete systems [8,9], impulsive systems [10] and so on.

简介：孙飞（1998），男，江苏常州，电气工程及其自动化，2017 级，区间观测器设计。

If we consider a linear discrete system without disturbance, i.e., $x(k+1) = Ax(k) + Bu(k)$, the task of IO design is to find a gain L such that the corresponding upper (or lower) error system $e^{+(-)}(k+1) = (A-LC)e^{+(-)}(k)$ is both positive and stable. Equivalently, it is desired that $A-LC$ is both non-negative and Schur stable. Whereas, it only requires that $A-LC$ is Schur stable in the context of conventional observers. From the aspect of computation, the non-negative of $A-LC$ is not easy to be verified by existing toolbox. Thus, the design of IO is much more complicated than that of conventional observer[11-12]. In order to overcome the drawback, the references [3,5,7,9] employed the coordinate transformation method to get more freedom of the construction of the IO. Actually, the IOs designed in these works are a class of asymptotical IOs.

The investigation of switched systems has drawn considerable attention in recent years [13-15]. Switched systems are ubiquitous in many practical systems, such as traffic networks [16], chemical engineering systems [17], circuit systems[18], etc. It is known that the works on IOs of switched systems are still challenging [19-22]. [19] and [20] designed the IOs for switched systems under the assumption that $A_i - L_iC_i$ is Metzler matrix. In order to improve the former results, [21] and [22] presented the IO design approaches for uncertain discrete-time and continuous-time switched systems by using coordinate transformation respectively. Recently, [23] improved the result of [21] by using the zonotope method, [24] designed an asynchronous IO for switched systems. In addition, the functional IO for linear discrete-time systems with disturbances and fixed-time observer for switched systems were also studied by Che et al.[25] and Gao et al.[26] respectively. However, the finite-time IO(FTIO) for discrete-time switched systems has not been reported.

Motivated by above discussion, the goal of this paper is to design FITO for discrete-time switched systems. In the light of definition of finite-time stability [27-29], the observer gains are selected such that the observation errors are bounded in finite time. The contribution of this work can be concluded as the following aspects:

(1) The bounds of the original systems can be recovered in a prescribed time interval.

(2) The existence conditions of the IO are derived by multiple linear copositive Lyapunov function (MLCLF), which is a useful tool when dealing with switched systems.

(3) The derived conditions are given by linear programming (LP) constraints which are more tractable than linear matrix inequalities.

The rest of paper is organized as follows. In Section 2, the plant as well as the structure of FTIO is given. In Section 3, by using MLCLF, sufficient conditions in the forms of LP are presented. Finally, in Section 4, one example is simulated to demonstrate the validity of the proposed method.

Notations: Throughout this paper, x^T is the transposition of the vector x, and A^T is the transposition of the matrix A. $\|x\|_1$ represents the 1-norm of the vector x. The symbols $\leq$, $<$, $\geq$ and $>$ are understood component-wise for any vector or matrix. E^+ represents $\max\{E, O\}$, where O is the zero matrix, and E^- equals to $E^+ - E$. $\overline{\kappa}(x)$ and $\underline{\kappa}(x)$ denote the maximum valueand the minimum value of the elements of x respectively.

Problem Statement and Preliminary

Consider the following plant:

$$\begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) + E_{\theta(k)}w(k), \\ y(k) = C_{\theta(k)}x(k), \\ \underline{x}(0) \leq x(0) \leq \overline{x}(0). \end{cases} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$ and $y(k) \in R^q$ are the state, input and output, respectively. $w(k) \in R^r$ is the perturbation with $w^- \leq w(k) \leq w^+$, where w^-, w^+ are given vectors. $\theta(k)$ is the switching signal and

$\theta(k) \in S = \{1, 2, \cdots, N\}$. $A_{\theta(k)} \in R^{n\times n}$, $B_{\theta(k)} \in R^{n\times m}$, $E_{\theta(k)} \in R^{n\times r}$ and $C_{\theta(k)} \in R^{q\times n}$ are given matrices, $\underline{x}(0) \in R^n$ and $\overline{x}(0) \in R^n$ are known vectors. For simplicity, $\theta(k)$ is short for θ, and the system (1) becomes

$$\begin{cases} x(k+1) = A_\theta x(k) + B_\theta u(k) + E_\theta w(k), \\ y(k) = C_\theta x(k), \\ \underline{x}(0) \le x(0) \le \overline{x}(0). \end{cases} \tag{2}$$

Definition 1.*[2] The interval frame* $\{\overline{x}(k), \underline{x}(k)\}$ *is called an asymptotical IO for (1) if for* $\forall k > 0$

$$\begin{cases} \lim_{k\to\infty} \| \overline{x}(k) - x(k) \|_1 = \alpha, \\ \lim_{k\to\infty} \| x(k) - \underline{x}(k) \|_1 = \beta, \end{cases}$$

where α *and* β *are positive constants.*

Remark 1. Definition 1 is just the extension of Definition 2 in [2] when the discrete case is discussed. In the light of positive switched system [30,31], we use the MLCLF to analyze stability of the error, thus 1-norm is employed to describe the bound of the error in this paper.

Definition 2. *The interval frame* $\{\overline{x}(k), \underline{x}(k)\}$ *is called a FTIO if there exists* K>0 *such that*

$$\| \overline{x}(0) - x(0) \|_1 \le \alpha_1 \Rightarrow \| \overline{x}(k) - x(k) \|_1 \le \alpha_2, \forall k \in [0, K], \tag{3}$$

$$\| x(0) - \underline{x}(0) \|_1 \le \beta_1 \Rightarrow \| x(k) - \underline{x}(k) \|_1 \le \beta_2, \forall k \in [0, K], \tag{4}$$

where α_1, α_2, β_1 *and* β_2 *are positive constants, and* $\alpha_1 < \alpha_2$, $\beta_1 < \beta_2$.

Remark 2. *From the aspect of application, the FTIO is necessary. Definition 1 is known to characteristic of the error in infinite time interval, but Definition 2 is with respect to the boundedness of the error in finite time. In fact, a FTIO may not be an asymptotical IO, and vice versa.*

We now extend the results of [32] to positive switched systems. The system is considered as:

$$\begin{cases} x(k+1) = M_\theta x(k) + f_\theta(k), \\ x(0) = x_0 \ge 0, \end{cases} \tag{5}$$

where $x(k) \in R^n$, θ is the switched law. $M_\theta \in R^{n\times n}$ is the constant matrix, and $f_\theta(k) \in R^n \ge 0$.

Lemma 1. *The system (5) is positive if and only if the matrix* $M_\theta \ge 0$.

Then, we construct the IO for the system (2), which has the following form:

$$\begin{cases} \overline{x}(k+1) = A_\theta \overline{x}(k) + B_\theta u(k) + E_\theta^+ w^+ - E_\theta^- w^- \\ \qquad + L_\theta(y(k) - C_\theta \overline{x}(k)), \\ \underline{x}(k+1) = A_\theta \underline{x}(k) + B_\theta u(k) + E_\theta^+ w^- - E_\theta^- w^+ \\ \qquad + L_\theta(y(k) - C_\theta \underline{x}(k)), \\ \overline{x}(0) = x^+(0), \\ \underline{x}(0) = x^-(0). \end{cases} \tag{6}$$

Let $\overline{x}(k) \leq x(k) \leq \underline{x}(k)$ and $e^-(k) = x(k) - \underline{x}(k)$. Comparing (6) with (2), we have

$$\begin{cases} e^+(k+1) = (A_\theta - L_\theta C_\theta) e^+(k) + \Gamma_\theta^+ - E_\theta w(k) \\ e^-(k+1) = (A_\theta - L_\theta C_\theta) e^-(k) + E_\theta w(k) - \Gamma_\theta^-, \\ e^-(0) \geq 0, e^+(0) \geq 0, \end{cases} \tag{7}$$

where $\Gamma_\theta^+ = E_\theta^+ w^+ - E_\theta^- w^-$, and $\Gamma_\theta^- = E_\theta^+ w^- - E_\theta^- w^+$.

Definition 3.[27-28] *Consider the system (7). Let* c_1, c_2, c_3, c_4, K, h *be positive constants with* $c_1 < c_2$ *and* $c_3 < c_4$. *If* $\forall w(k): \sum_{k=0}^{\cdot\cdot} \| w(k) \|_1 \leq h$

$$\| e^+(0) \|_1 < c_1 \Rightarrow \| e^+(k) \|_1 < c_2, \forall\ k \leftarrow [0, K], \tag{8}$$

$$\| e^-(0) \|_1 \leq c_3 \Rightarrow \| e^-(k) \|_1 \leq c_4, \forall\ k \in [0, K], \tag{9}$$

then the upper and lower error system (7) is finite-time bound (FTB).

Definition 4.[33] *Denote the switching number of* θ *on the interval* $[l_1, l_2)$ *by* $N_\theta(l_1, l_2)$. *If*

$$N_\theta(l_1, l_2) \leq N_0 + (l_2 - l_1) / \tau^*$$

holds for given $N_0 \geq 0$ *and* $\tau^* > 0$, *then* τ^* *is the average dwell time (ADT).* In what follows, N_0 is supposed to be 0.

Lemma 2.[34] *Let* $\Theta(k) \in R^n$ *with* $\Theta^-(k) \leq \Theta(k) \leq \Theta^+(k)$, *then the following holds*

$$W^+ \Theta^-(k) - W^- \Theta^+(k) \leq W \Theta(k) \leq W^+ \Theta^+(k) - W^- \Theta^-(k),$$

where $W \in R^{m \times n}$ *is any given constant matrix.*

Main Result

In this section, the performance analysis of the error system (7) is presented.

Theorem 1. *Let* $\nu > 1$ *and* $\text{ñ} > 1$ *be two constants. If there are vectors* $v_i \in R^n > 0$, $v_j \in R^n > 0$, $z_i \in R^q$, *and prescribed vector* $\xi_i \in R^n \neq 0$ *for* $i, j \in S$, $i \neq j$ *such that*

$$(A_i^T - \nu I)v_i + C_i^T z_i < 0, \tag{10}$$

$$v_i \leq \text{ñ} v_j, \tag{11}$$

$$\xi_i^T v_i(\xi_i^T v_i A_i + \xi_i z_i^T C_i) \geq 0, \tag{12}$$

and the observer gain L_i *has the following form*

$$L_i = -\frac{\xi_i z_i^T}{\xi_i^T v_i}, \tag{13}$$

then the upper and lower error system (7) satisfies the property of positive and FTB. Furthermore, denote that

$$\max_{i\in S}\{(\Gamma_i^+)^T v_i\} = \lambda, \tag{14}$$

$$\max_{i\in S}\{(\Gamma_i^-)^T v_i\} = \delta, \tag{15}$$

$$\max_{i\in S}\{\| E_i^T v_i \|_1\} = \gamma, \tag{16}$$

where λ , δ *and* $\gamma > 0$ *are constants, then ADT satisfies*

$$\tau^* \geq \max\{\frac{K \ln \text{耨}}{\ln \mu_1 - \ln \zeta_1 - K \ln \nu}, \frac{K \ln}{\ln \mu_2 - \ln \zeta_2 - K \ln \nu}\}, \tag{17}$$

where $\mu_1 = c_2 l_1$, $\mu_2 = c_4 l_1$, $\zeta_1 = c_1 l_2 + \gamma h + |\lambda| K$, $\zeta_2 =$ $c_3 l_2 + \gamma h + |\delta| K$ *with* $l_1 = \min_{i\in S}\{\underline{\kappa}(v_i)\}$, $l_2 = \overline{\kappa}(v_{\theta(0)})$, $\mu_1 > \zeta_1 \nu^K$ *and* $\mu_2 > \zeta_2 \nu^K$.

Proof. From Definition 2 and Definition 3, the following proof will be divided into steps:

Firstly, by (13), we obtain

$$A_i - L_i C_i = A_i + \frac{\xi_i z_i^T}{\xi_i^T v_i} C_i, \tag{18}$$

which follows from (12) that

$$A_i - L_i C_i = A_i + \frac{\xi_i z_i^T}{\xi_i^T v_i} C_i \geq 0. \tag{19}$$

By Lemma 2, we have $\Gamma_i^+ - E_i w(k) \geq 0$ and $E_i w(k) - \Gamma_i^- \geq 0$. That means $e^-(0) \geq 0$ and $e^+(0) \geq 0$, so that the residual error of the system is bounded by the designed observer. Thus, in view of Lemma 1, the error system (7) is positive. We have

$$\underline{x}(k) \leq x(k) \leq \overline{x}(k).$$

Secondly, the following error system is considered

$$\begin{cases} e^+(k+1) = (A_\theta - L_\theta C_\theta)e^+(k) + \Gamma_\theta^+ - E_\theta w(k), \\ e^+(0) \geq 0. \end{cases} \tag{20}$$

Let $\{k_p, p = 1, 2, \cdots\}$ with $0 < k_1 < k_2 < \cdots$ be the switching time sequence. If $\theta(k_s) = i \in S$, then the MLCLF is chosen as follows:

$$V_i(K) = (e^+(K))^T v_i, i \in S. \tag{21}$$

When $K \in [k_p, k_{p+1})$, taking the backward difference of $V_i(K)$ yields

$$\begin{aligned} \nabla V_i(K) &= V_i(K) - V_i(K-1) \\ &= (e^+(K-1))^T (A_i^T - C_i^T L_i^T) v_i - (e^+(K-1))^T v_i \\ &+ (\Gamma_i^+)^T v_i - (w(K-1))^T E_i^T v_i. \end{aligned} \tag{22}$$

Substituting (13) into (22) results in

$$\begin{aligned} \nabla V_i(K) = (e^+(K-1))^T (A_i^T v_i + C_i^T z_i - v_i) \\ + (\Gamma_i^+)^T v_i - (w(K-1))^T E_i^T v_i. \end{aligned} \tag{23}$$

By (10), (14) and (16), we can obtain

$$\begin{aligned} \nabla V_i(K) &\leq (\nu - 1)(e^+(K-1))^T v_i + \lambda + \| w(K-1) \|_1 \| E_i^T v_i \|_1 \\ &\leq (\nu - 1) V_i(K-1) + \lambda + \gamma \| w(K-1) \|_1, \end{aligned} \tag{24}$$

i.e.,

$$V_i(K) \leq \nu V_i(K-1) + \lambda + \gamma \| w(K-1) \|_1 . \tag{25}$$

For the interval $[k_p, K)$, it is concluded that

$$V_i(K) \leq \nu^{K-k_p} V_i(k_p) + \gamma \sum_{s=k_p}^{K-1} \nu^{K-1-s} \| w(s) \|_1 + \lambda \sum_{s=k_p}^{K-1} \nu^{K-1-s}. \tag{26}$$

Suppose that $\theta(k_{p-1}) = j$, it follows from (11) and (26) that

$$V_i(K) \le \varrho \nu^{K-k_p} V_j(k_p) + \gamma \sum_{s=k_p}^{K-1} \nu^{K-1-s} \| w(s) \|_1 + \lambda \sum_{s=k_p}^{K-1} \nu^{K-1-s}. \tag{27}$$

Repeating (26) and (27) yields

$$\begin{aligned} V_i(K) &\le \varrho \nu^{K-k_p} V_{\theta(k_{p-1})}(k_p) + \gamma \sum_{s=k_p}^{K-1} \nu^{K-1-s} \| w(s) \|_1 + \lambda \sum_{s=k_p}^{K-1} \nu^{K-1-s} \\ &\le \varrho^{N_\theta(0,K)} \nu^K V_{\theta(0)}(0) + \gamma \sum_{s=0}^{K-1} \varrho^{N_\theta(s,K)} \nu^{K-1-s} \| w(s) \|_1 \\ &+ \lambda \sum_{s=0}^{K-1} \varrho^{N_\theta(s,K)} \nu^{K-1-s} \\ &\le \varrho^{N_\theta(0,K)} \nu^K V_{\theta(0)}(0) + \gamma \sum_{s=0}^{K-1} \varrho^{N_\theta(s,K)} \nu^{K-1-s} \| w(s) \|_1 \\ &+ |\lambda| \sum_{s=0}^{K-1} \varrho^{N_\theta(s,K)} \nu^{K-1-s}. \end{aligned} \tag{28}$$

From Definition 4, we have $N_\theta \le N_0 + \frac{K}{\tau^*} = \frac{K}{\tau^*}$. Since $\nu > 1$ and $\sum_{s=0}^{K-1} \| w(s) \|_1 \le h$, the above equality (28) becomes

$$\begin{aligned} V_i(K) &\le \varrho^{N_\theta(0,K)} \nu^K (V_{\theta(0)}(0) + \gamma h + |\lambda| K) \\ &\le \varrho^{\frac{K}{\tau^*}} \nu^K (V_{\theta(0)}(0) + \gamma h + |\lambda| K). \end{aligned} \tag{29}$$

It is the fact that

$$\begin{cases} V_i(K) = (e^+(K))^T v_i \ge l_1 \| e^+(K) \|_1, \\ V_{\theta(0)}(0) = (e^+(0))^T v_{\theta(0)} \le l_2 \| e^+(0) \|_1 . \end{cases} \tag{30}$$

Substituting (30) into (29) results in

$$l_1 \| e^+(K) \|_1 \le \tilde{n}^{\frac{K}{\tau^*}} \nu^K (l_2 \| e^+(0) \|_1 + \gamma h + |\lambda| K). \tag{31}$$

In view of (17) and $\tilde{n} > 1$, (31) implies that

$$\| e^+(K) \|_1 \le \frac{\mu_1}{l_1 \zeta_1} (l_2 \| (e^+(0)) \|_1 + \gamma h + |\lambda| K). \tag{32}$$

When $\| e^+(0) \|_1 \le c_1$, it is deduced from (32) that

$$\| e^+(K) \|_1 \le \frac{\mu_1}{l_1 \zeta_1} (c_1 l_2 + \gamma h + |\lambda| K). \tag{33}$$

Considering the expressions $\mu_1 = c_2 l_1, \zeta_1 = c_1 l_2 + \gamma h + |\lambda| K$, (33) means

$$\| e^{+}(K) \|_1 \leq c_2. \tag{34}$$

Let us turn to the following error system:

$$\begin{cases} e^{-}(k+1) = (A_\theta - L_\theta C_\theta)e^{-}(k) + E_\theta w(k) - \Gamma^{-}_\theta, \\ e^{-}(0) \geq 0. \end{cases} \tag{35}$$

The MLCLF candidate is chosen as

$$\tilde{V}_i(K) = (e^{-}(K))^T v_i, \ i \in S. \tag{36}$$

By the same treatment as that in the upper error system, one can get

$$\tilde{V}_i(K) \leq \tilde{n}^{N_\theta(0,K)} \nu^K (\tilde{V}_{\theta(0)}(0) + \gamma h + |\delta| K). \tag{37}$$

By (17), we have

$$\| e^{-}(K) \|_1 \leq \frac{\mu_2}{l_1 \zeta_2} (l_2 \| (e^{-}(0)) \|_1 + \gamma h + |\delta| K). \tag{38}$$

In view of $\mu_2 = c_4 l_1$, $\zeta_2 = c_3 l_2 + \gamma h + |\delta| K$, when $\| e^{-}(0) \|_1 \leq c_3$, we obtain

$$\| e^{-}(K) \|_1 \leq c_4. \tag{39}$$

In view of Definition 3, the system (7) satisfies the property of FTB. Thus, we can conclude that (6) is a FTIO for the system (2).

Remark 3. *The constraints* (10)-(12) *are the existence conditions of the finite-time IO* (6)*, while the expressions* (14)-(16) *are used for the estimation of the boundness of the error. However, the feasible solutions can not be solved from the conditions* (10)-(12) *by Matlab because of the term* $(\xi_i^T v_i)^2$ *in* (12)*. Thus, we need derive the equivalent forms instead of* (10)-(12)*.*

We now give the following theorem, which is necessary from the aspect of computation.

Theorem 2. *Let* $\nu > 1$ *and* $\tilde{n} > 1$ *be two constants. Assume that* L_i *is determined by (13) and* τ^* *satisfies (17). If there exist vectors* $v_i \in R^n > 0$, $v_j \in R^n > 0$, $z_i \in R^q$*, and prescribed vector* $\xi_i \in R^n \neq 0$ *for* $i, j \in S$, $i \neq j$ *such that*

$$(A_i^T - \nu I)v_i + C_i^T z_i < 0, \tag{40}$$

$$v_i \leq \tilde{n} v_j, \tag{41}$$

$$\xi_i^T v_i > 0, \tag{42}$$

$$\xi_i^T v_i A_i + \xi_i z_i^T C_i \geq 0, \tag{43}$$

or

$$(A_i^T - \nu I)v_i + C_i^T z_i < 0, \tag{44}$$

$$v_i \leq \tilde{n} v_j, \tag{45}$$

$$\xi_i^T v_i < 0, \tag{46}$$

$$\xi_i^T v_i A_i + \xi_i z_i^T C_i \leq 0. \tag{47}$$

the upper and lower error system (7) *is positive and FTB.*

Proof. Let us consider the bilinear constraint (12). If $\xi_i^T v_i > 0$, then (12) means that $\xi_i^T v_i A_i + \xi_i z_i^T C_i \geq 0$. If $\xi_i^T v_i < 0$, then (12) implies that $\xi_i^T v_i A_i + \xi_i z_i^T C_i \leq 0$. Thus, the conditions (40)-(43) or (44)-(47) indicates (10)-(12).

Remark 4. *In order to design the IO (6) and give the estimation of the error, we employ the following steps:*
Step1: Solve z_i, v_i *(40)-(43) or (44)-(47) by Linprog in Matlab;*
Step2: Determine L_i *by (13) and* λ, δ, γ *by (14)-(16) respectively;*
Step3: Compute μ_1, ζ_1, μ_2 *and* ζ_2 *with* $\mu_1 > \zeta_1 \nu_1^K$ *and* $\mu_2 > \zeta_2 \nu_2^K$*;*
Step4: Estimate c_2 *and* c_4.

From Remark 4, c_2 and c_4 are only bounded constants when we obtain the feasible solutions from the sufficient conditions. From the aspect of practice, c_2 and c_4 are both expected to be minimal. Thus, the following theorem is stated.

Theorem 3. *If the following convex optimization problem can be solved*

$$\begin{cases} \min \ c_2, c_4 \\ subject \ to: \\ (A_i^T - \nu I)v_i + C_i^T z_i < 0, \\ v_i \leq \tilde{n} v_j, \\ \xi_i^T v_i > 0, \\ \xi_i^T v_i A_i + \xi_i z_i^T C_i \geq 0, \end{cases} \tag{48}$$

or

$$\begin{cases} \min \ c_2, c_4 \\ subject \ to: \\ (A_i^T - \nu I)v_i + C_i^T z_i < 0, \\ v_i \leq \tilde{n} v_j, \\ \xi_i^T v_i < 0, \\ \xi_i^T v_i A_i + \xi_i z_i^T C_i \leq 0, \end{cases} \tag{49}$$

then the IO (6) is an optimal FTIO.

Remark 5. *By Theorem 1,* c_2 *is dependent on* γ, δ, l_2 *and* c_1*, while* c_4 *is dependent on* γ, δ, l_2 *and* c_3. *It is also the fact that* γ, λ, δ, l_2 *are determined once* v_i *is fixed. In order to minimize the error estimation,* c_2 *should be chosen as small as possible by computing (48) or (49), and it is the same with* c_4. *A suggested algorithm is given as follows: The first step updates all the parameters such as* ν, ñ *by the path-following method proposed in [35]. The second step fixes the parameters* ν, ñ *to solve* v_i. *We repeat the above two steps until* c_2 *and* c_4 *reach the minimum values.*

Numerical Example

Consider the system (2) with two modes, and the system matrices are given as:

$$A_1 = \begin{bmatrix} 1.2 & 2.2 \\ 1.8 & 1.6 \end{bmatrix},\ A_2 = \begin{bmatrix} 1.5 & 1.6 \\ 2.5 & 2.3 \end{bmatrix},\ B_1 = \begin{bmatrix} 1.3 & 1.2 \\ 1.5 & 1.7 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 2.1 & 1.9 \\ 1.8 & 1.4 \end{bmatrix},\ C_1 = \begin{bmatrix} 1.5 & 1.4 \\ 1.1 & 1.2 \end{bmatrix},\ C_2 = \begin{bmatrix} 1.3 & 1.6 \\ 1.5 & 1.7 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.7 & -1 \\ -0.8 & 0.5 \end{bmatrix},\ E_2 = \begin{bmatrix} -0.6 & 0.5 \\ 0.4 & -0.9 \end{bmatrix}.$$

For the purpose of simulation, $u(k)$, $\omega(k)$ and x_0, x_0^+ and x_0^- are chosen as follows:

$$u(k) = \begin{bmatrix} \sin^2 k \\ \cos 2k \end{bmatrix},\ \omega(k) = \begin{bmatrix} 0.1\cos^2 k \\ 0.1\sin k \end{bmatrix},\ x_0 = \begin{bmatrix} 5 \\ 10 \end{bmatrix},$$

$$x_0^+ = \begin{bmatrix} 10 \\ 20 \end{bmatrix},\ x_0^- = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Let $\xi_1 = [1;2]$, $\xi_2 = [2;1]$, $K = 4$. By solving the sufficient conditions of Theorem 3, we have

$$v_1 = \begin{bmatrix} 49.656 \\ 52.4553 \end{bmatrix},\ z_1 = \begin{bmatrix} -58.4805 \\ -23.2261 \end{bmatrix},\ v_2 = \begin{bmatrix} 53.543 \\ 40.9385 \end{bmatrix},$$

$$z_2 = \begin{bmatrix} -19.9887 \\ -44.6094 \end{bmatrix},\ \nu = 1.775,\ \text{ñ} = 1.3.$$

Thus, we can determine the observer gain

$$L_1 = \begin{bmatrix} 0.3784 & 0.1503 \\ 0.7567 & 0.3005 \end{bmatrix},\ L_2 = \begin{bmatrix} 0.2701 & 0.6027 \\ 0.135 & 0.3014 \end{bmatrix},$$

the ADT $\tau^* \geq 1.8$, $c_2 \leq 24.8142$ and $c_4 \leq 23.3343$. In the sequel, we use Simulink in Matlab to complete the simulation. The switching signal $\theta(k)$ is depicted in Fig.1. The performance of the IO (6) is given in Fig.2. We can see that $\overline{x}_1(k)$-$x_1(k)$ and $x_1(k)$-$\underline{x}_1(k)$ are always positive and bounded. And it is the same in Fig.3. The response of errors is presented in Fig.4 and Fig.5, where the errors are bounded within 1.5s and 4s. Thus, the errors are FTB.

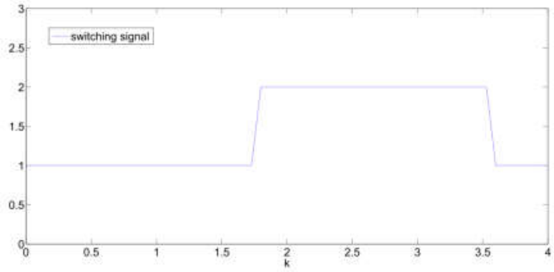

Figure 1. Switching signal $\theta(k)$ with ADT property

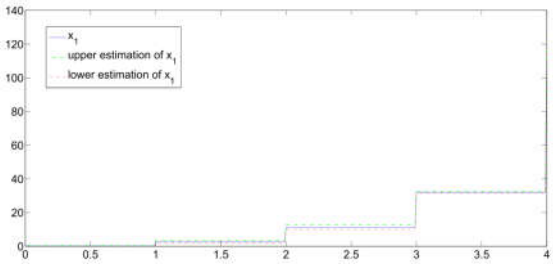

Figure 2. Response of $x_1(k)$, $\overline{x}_1(k)$ and $\underline{x}_1(k)$

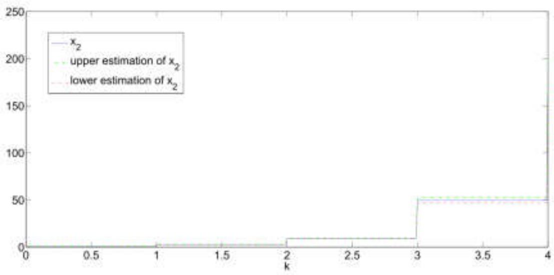

Figure 3. Response of $x_2(k)$, $\overline{x}_2(k)$ and $\underline{x}_2(k)$

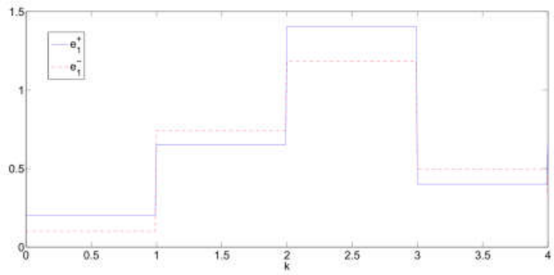

Figure 4. Response of the errors $e_1^+(k)$, $e_1^-(k)$

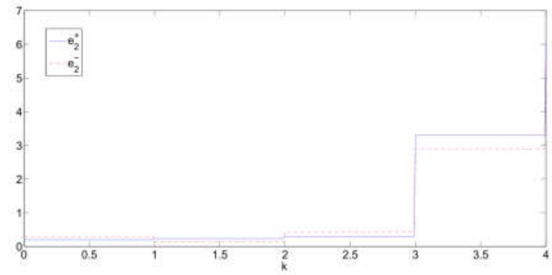

Figure 5. Response of the errors $e_2^+(k)$, $e_2^-(k)$

Conclusion

A FTIO design framework for discrete-time switched systems subjected to disturbances is presented. The framework of the FTIO is constructed and the stability conditions are obtained by using MLCLF. Different from the works herein, such as [19-22], all the conditions established are given by the forms of LP. Besides, the errors can be kept in a bounded neighborhood for a given time interval. In the future, the FTIO design method for nonlinear switched systems will be investigated.

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